Observation of strongly non-Gaussian statistics for random sea surface gravity waves in wave flume experiments

M. Onorato, A. R. Osborne, and M. Serio
Dipartimento di Fisica Generale, Università di Torino, Via Pietro Giuria 1, 10125 Torino, Italy

L. Cavaleri
ISMAR, S. Polo 1364, 30125 Venezia, Italy

C. Brandini
La.M.M.A., Regione Toscana, Via Madonna del Piano, 50019 Sesto Fiorentino, Italy

C. T. Stansberg
Norwegian Marine Technology Research Institute A.S (Marintek), P.O. Box 4125 Valentinlyst, N-7450 Trondheim, Norway
(Received 5 May 2004; published 30 December 2004)

We study random surface gravity wave fields and address the formation of large-amplitude waves in a laboratory environment. Experiments are performed in one of the largest wave tank facilities in the world. We present experimental evidence that the tail of the probability density function for wave height strongly depends on the Benjamin-Feir index (BFI)—i.e., the ratio between wave steepness and spectral bandwidth. While for a small BFI the probability density functions obtained experimentally are consistent with the Rayleigh distribution, for a large BFI the Rayleigh distribution clearly underestimates the probability of large events. These results confirm experimentally the fact that large-amplitude waves in random spectra may result from the modulational instability.

DOI: 10.1103/PhysRevE.70.067302 PACS number(s): 47.20.Ma, 47.35.+i, 92.10.Hm

The determination of the probability density function of wave heights for a system of a large number of random waves is definitely a task of major importance from both theoretical and applicative points of view. For linear waves Longuett-Higgins [1] showed that, if the wave spectrum is narrow banded and if the phases of the Fourier components of the surface elevation are distributed uniformly, then the probability distribution of crest-to-trough wave heights is given by the Rayleigh distribution. After the pioneering work by Longuett-Higgins [1], the validity of the Rayleigh distribution for wave heights has been widely investigated. The distribution was found to agree well with many field observations [2] even though the frequency spectrum was not always as narrow and the steepness was not as small as required by the theory. Nevertheless, in the last few years, times series recorded in the ocean have shown that extreme wave events can appear on the surface of the ocean (the most striking is the one recorded in the North Sea in 1995, where a wave of 26 m height was measured; see, for example [3]). Faced with such measurements, two questions can be naturally formulated: (i) What is the physical mechanism of formation of these waves? (ii) Do such large-amplitude waves obey a different distribution than Rayleigh?

Even though a number of physical mechanisms for the formation of large-amplitude waves have been identified [linear superposition [1], the wave-current interaction (see [4,5])], and the modulational instability [3,6,7], (known also as Benjamin-Feir instability [8,9]), it should be stated that not much theoretical and experimental progresses have been made concerning the resulting statistical properties of the surface elevation. More in particular, the relation between the various sea states and the probability density function has not been clearly identified. Onorato *et al.* [11], based on dimensional considerations for the nonlinear Schrödinger (NLS) equation, have pointed out that the ratio between the wave steepness and the spectral bandwidth [after [13] this ratio is known as the Benjamin-Feir index (BFI)] plays an important role in the appearance of large amplitude waves. Numerical simulations of envelope equations (nonlinear Schrödinger and higher-order equations) characterized by initial conditions provided by a Joint North Sea Wave Project (JONSWAP) spectrum with random phases showed that the probability density function of wave heights strongly depends on the BFI (see [10,11]): for small values of the BFI, the Rayleigh distribution approximates the numerical data rather well, but for large values of the BFI the Rayleigh distribution clearly underestimates the tail of the probability density function obtained from numerical simulations. This departure from the Rayleigh distribution was attributed to the

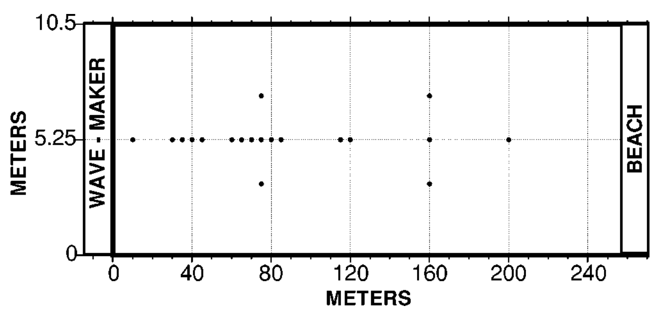

FIG. 1. Schematic of the wave tank facility at Marintek. Solid circles indicate the position of the probes.

TABLE I. Parameters of the three different experiments performed at Marintek.

$H_s(m)$	ϵ	$\Delta f/f_0$	BFI
0.11	0.098	0.28	0.2
0.14	0.125	0.09	0.9
0.16	0.142	0.08	1.2

Benjamin-Feir instability mechanism—i.e., to the formation of unstable, coherent modes which characterize the evolution of the NLS equation [12]. According to these results, the BFI could be thought of as a measure of the importance of the modulational instability in random spectra. Similar conclusions have been obtained by Janssen [13] who, starting from the Zakharov equation, has developed a kinetic equation that takes into account quasiresonant interactions—i.e., the modulational instability; he was able to compute from the developed theory statistical quantities of the surface elevation such as, for example, the kurtosis. He found out that, if the ratio between the steepness and the spectral bandwidth is large, the Gaussian distribution underestimates the tails of the probability density function for the surface elevation. It is important to stress here that all the cited results have been reached numerically and theoretically from simplified models but, until now, they had never been verified experimentally.

In this Brief Report we present the first experimental evidence that the wave statistics depend on the BFI. Our main goal here is to give some experimental support to the numerical and theoretical work performed in recent years that suggests the idea that the modulational instability (Benjamin-Feir instability) can be responsible for the formation of freak waves. More in particular our intent here is to underline the importance of the BFI for the determination of the probability density function of wave heights.

The experiment was carried out in the long-wave flume at Marintek (see, e.g. [14], for the description of the tank). A sketch of the wave flume is given in Fig. 1. The length of the tank is 270 m and its width is 10.5 m. The wave surface elevation was measured simultaneously by 19 probes placed at different locations along the flume. Conditions at the wave maker were built as random wave signals characterized by a JONSWAP power spectrum. Signals driving the wave maker were prepared according to the "random realization approach" by using random spectral amplitudes as well as random phases. Three different JONSWAP spectra characterized by different BFI have been considered. In Table I we report the parameters—i.e., the significant wave height H_s—computed as 4 times the standard deviation of the free surface elevation, the steepness ϵ, and the relative frequency spectral bandwidth $\Delta f/f_0$, which characterized each JONSWAP spectrum. According to the definition (see [13]) the BFI is also computed as BFI$=\sqrt{2}\epsilon/(2\Delta f/f_0)$. Details of the experiments can be found in [15]. The three different experiments will be called BFI0.2, BFI0.9, and BFI1.2, with obvious meaning. The dominant frequency was selected to be $f_0=0.6667$ Hz. In order to have sufficiently good statistics, a large number of waves were recorded. Note that a large amount of data is of fundamental importance for the convergence of the tail of the probability density function for wave heights. Therefore for each type of spectrum, five different realizations with different sets of random phases have been performed. The duration of each experimental realization was 32 min. The total number of wave heights (counting

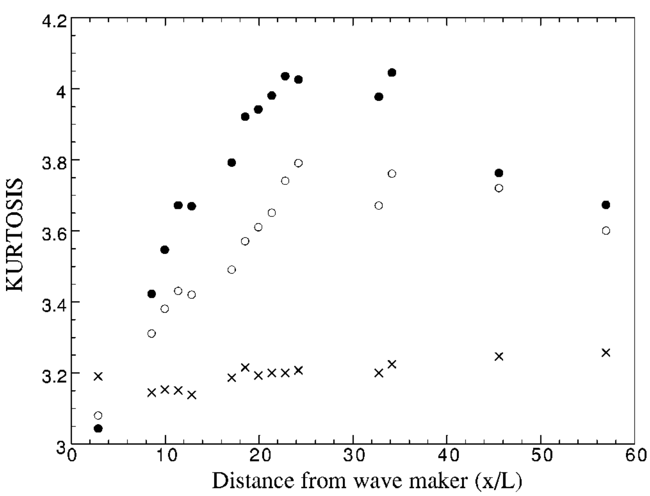

FIG. 2. Evolution of the kurtosis along the wave tank: BFI0.2, crosses; BFI0.9, open circles; BFI1.2, solid circles. The horizontal axis has been nondimensionalized with the characteristic wavelength computed using the linear dispersion relation.

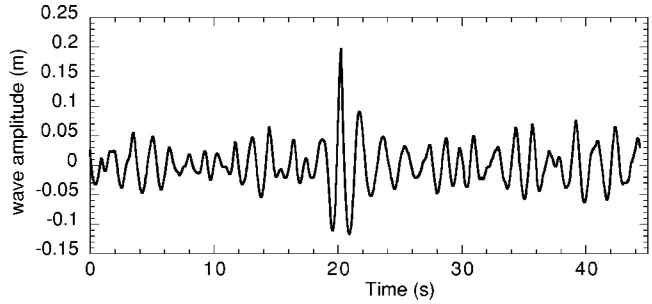

FIG. 3. Time series recorded for BFI=1.2 at probe 14 showing a large-amplitude wave.

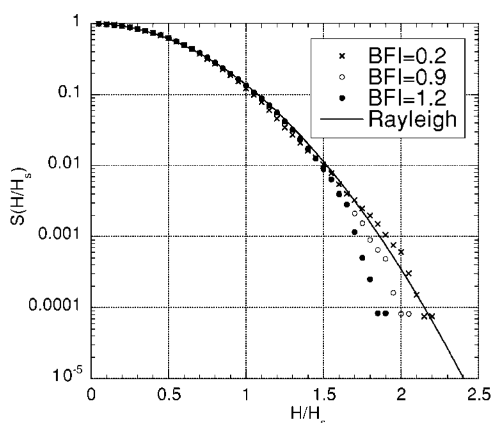

FIG. 4. Survival function at $x/L=2.8$ for BFI0.2 (crosses), BFI0.9 (open circles), and BFI1.2 (solid circles).

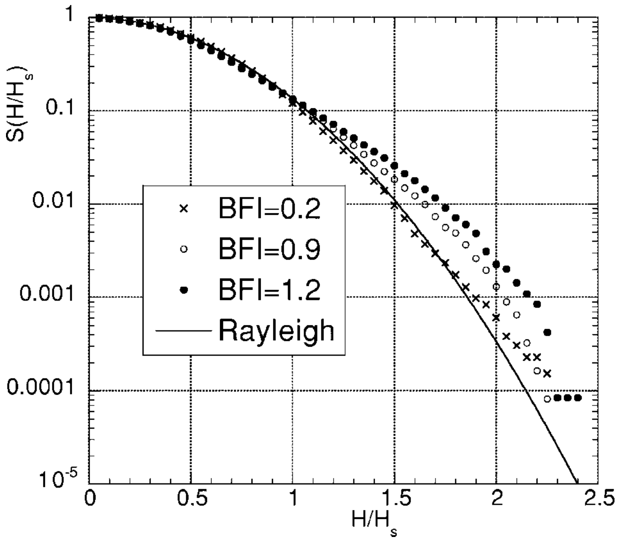

FIG. 5. Survival function at $x/L=18.5$ for BFI0.2 (crosses), BFI0.9 (open circles), and BFI1.2 (solid circles).

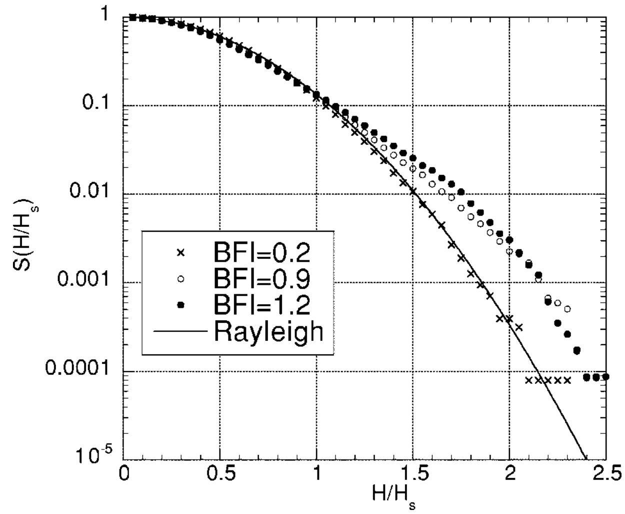

FIG. 6. Survival function at $x/L=32.7$ for BFI0.2 (crosses), BFI0.9 (open circles), and BFI1.2 (solid circles).

both up-crossing and down-crossing) recorded for each spectral shape at each probe was about 12 800 waves.

We first consider the behavior of some statistical quantities that can give an indication of the presence of extreme events in the time series. In particular we consider the fourth-order moment of the probability density function, the kurtosis, which gives an indication of the importance of the tail of the distribution function. We recall that for a Gaussian distribution the value of the kurtosis is 3, while larger values of kurtosis indicate the presence of extreme events. In Fig. 2 we show the kurtosis for the three experiments as a function of the distance from the wave maker. The horizontal axes have been nondimensionalized using a wavelength corresponding to the peak period at the wave maker: for $T=1.5$ s, $L=3.51$ m. First of all it should be noted from the figure that the kurtosis is always greater than the Gaussian prediction. For larger BFI (BFI=0.9 and BFI=1.2) the kurtosis grows very rapidly and reaches its maximum between 25 and 30 wavelengths from the wave maker. For the smallest value of the BFI considered, it is shown that the kurtosis is almost constant with a mean value close to 3.2. This result suggests a significant dependence on the BFI of the statistical properties of surface gravity waves. Before discussing the statistical properties of wave heights we show from our data just an example of the type of event that causes the kurtosis to depart from its Gaussian value. In Fig. 3 we show a typical large amplitude wave that characterizes our experimental time series. Those kind of events are much more frequent for BFI=0.9 and BFI=1.2 with respect to the case of BFI=0.2.

We now discuss the behavior of the survival function for wave heights, considering all together zero up-crossing and down-crossing wave heights. We compare our experimental results with the survival function obtained for the Rayleigh distribution—i.e., $\exp[-2(H/H_s)^2]$. We compare the survival function for the three different experiments at the same distance from the wave maker. In Fig. 4 we show the survival function at the first probe, $x/L=2.8$. We recall that the wave field has been generated at the wave maker as a linear superposition of random waves; therefore, we expect that at a few wavelengths the wave height should be described approximately by the Rayleigh distribution. For larger values of the BFI, the Rayleigh distribution overestimates the experimental data for large waves; this is consistent with most of the observations (see [16] and comments in [17]).

We then consider the probe at $x/L=18.5$; see Fig. 5. While the data from BFI=0.2 are well described by the Rayleigh distribution, it is quite clear from the plot that the experimental data for BFI=0.9 and BFI=1.2 are substantially underestimated by the Rayleigh distribution. The curve for BFI=1.2 lies always above the one with BFI=0.9 and separates from the Rayleigh distribution at around $H/H_s=1$, which corresponds to a probability of 1/10 waves. A similar behavior is seen in Fig. 6 at $x/L=32.7$.

The results shown give a clear experimental evidence that the BFI is an interesting parameter that affects the tail of the probability density function of wave heights. Starting with an initial spectrum characterized by random phases with a statistics that is Gaussian, our experimental data show that, if the BFI is sufficiently large, the time series recorded after about 25–30 wavelengths from the wave maker is characterized by large-amplitude events such as the one shown in Fig. 3. These events are the result of the Benjamin-Feir instability. In terms of the language of the nonlinear Schrödinger equation, this instability results in the so-called "breather modes"—i.e., nonlinear waves that grow, reach a maximum, and return to their initial state in a recurrent manner. In a nonlinear random wave field the presence of these waves is statistically significant, leaving clearly their signature in the survival function of wave heights. This phenomenological description, which has been conceived during many years of research with the envelope equations (see [6,7,10,11]), is very consistent with the behavior of the experimental data presented. Even if we have collected a large number of data, sufficient to observe a clear departure from the Rayleigh distribution, we believe that statistics computed on even larger data sets should be used in order to model the tail of the survival function. One of the limitations of our work that restrict us from extending our results in a straightforward manner to real wind waves is that our experiment has been performed in the case of infinite crested waves. For two-dimensional propagation, results on the statistical properties of wave amplitudes are much harder to obtain (see [18,19,20]) because numerical simulations become much more expensive and experimental work requires large basins with wave makers capable of generating waves in different directions.

This research has been supported by the Improving Human Potential–Transnational Access to Research Infrastructures Programme of the European Commission under Contract No. HPRI-CT-2001-00176. M.O., M.S., and A.R.O. acknowledge also the support from M.I.U.R. under PRIN research project.

[1] M. S. Longuet-Higgins, J. Mar. Res. **11**, 1245 (1952).
[2] M. D. Earle, J. Geophys. Res. **80**, 377–379 (1975).
[3] K. Trulsen and K. B. Dysthe, in *Proceedings of the 21st Symposium on Naval Hydrodynamics*, edited by Naval Studies Board (National Academy Press, Washington, DC, 1997), pp. 550–560.
[4] I. V. Lavrenov, Natural Hazards **17**, 117 (1998).
[5] B. S. White and B. Fornberg J. Fluid Mech. **255**, 113 (1998).
[6] K. B. Dysthe and K. Trulsen, Phys. Scr. **T82**, 48 (1999).
[7] A. R. Osborne, M., Onorato, and M. Serio, Phys. Lett. A **275**, 386 (2000).
[8] K. L. Henderson, D. H. Peregrine, and J. W. Dold, Wave Motion **29**, 341 (1999).
[9] M. P. Tulin and T. Waseda, J. Fluid Mech. **378**, 197 (1999).
[10] M. Onorato, A. R. Osborne, M. Serio, and T. Damiani, in *Proceedings of the Workshop Rogue Waves 2000, Brest, 2000*, edited by M. Olagnon and G. A. Athanassoulis (Ifremer, Brest, 2000), pp. 181–192.
[11] M. Onorato, A. R. Osborne, S. Bertone Serio, Phys. Rev. Lett. **86**, 5831 (2001).
[12] M. Onorato, A. R. Osborne, R. Fedele, and M. Serio, Phys. Rev. E **67**, 046305 (2003).
[13] P. A. E. M. Janssen, J. Phys. Oceanogr. **33**, 863 (2003).
[14] C. T. Stansberg, Internal Report 490030.01 Marintek, Sintef Group, Trondheim, 1993.
[15] M. Onorato, A. R. Osborne, M. Serio, L. Cavaleri, C. Brandini, and C. T. Stansberg (unpublished).
[16] G. Z. Forristall, J. Geophys. Res. **83**, 2553 (1978).
[17] M. S. Longuet-Higgins, J. Geophys. Res. **85**, 1519 (1980).
[18] M. Onorato, A. R. Osborne, and M. Serio, Phys. Fluids **14**, L25 (2002).
[19] H. Socquet-Juglard, K. Dysthe, K. Trulsen, H. E. Krogstad, and J. Liu (unpublished).
[20] J. F. Willemsen, J. Geophys. Res. **107**, 3121 (2002).